

Scientific paper

Experimental Investigation on Shear Cracking Behavior in Reinforced Concrete Beams with Shear Reinforcement

Mohamed Zakaria¹, Tamon Ueda², Zhimin Wu³ and Liang Meng⁴

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Abstract

This paper presents an experimental investigation to clarify shear cracking behavior of reinforced concrete beams. The effects of the various influential parameters on the spacing between shear cracks and the relationship between shear crack width and stirrup strain at the intersection with shear cracks were carefully investigated. It was found that shear crack width proportionally increases with both the strain of shear reinforcement and with the spacing between shear cracks. Greater diagonal crack spacings were found in larger beams and hence resulted in wider shear crack width. The test results also revealed that shear reinforcement characteristics (side concrete cover to stirrup, stirrup spacing and/or stirrup configuration) and longitudinal reinforcement ratio play a critical role in controlling the diagonal crack spacings and openings. It was illustrated that the distance of shear crack from the crack tip and the intersection with the nearest reinforcement can significantly affect the variation of shear crack width along the same shear crack. Conversely, the loading paths (loading, unloading and reloading paths) show an insignificant effect on shear crack width-stirrup strain relationship. Finally, the experimental results presented are useful information for the development of a rational shear crack displacement prediction method in existing design codes.

1. Introduction

Significant efforts have been put all over the world to clarify the problem of cracking and crack control in reinforced concrete (RC) members, which adversely affects structural performances in various ways such as serviceability and durability. In order to utilize the performance-based concept in design more efficiently, the clarification of cracking is quite important. Under the performance-based design, crack width is related to various required performances such as appearance, water-tightness and maintainability (reparability) of concrete structures. Crack width also affects durability which is required to keep the performance requirement by preventing degradation of various structural performances such as member strength and stiffness which would affect structural safety and serviceability.

Although the existing guidelines which are related to crack control in concrete structures provide some design formulae for crack width prediction, most of them were originally developed for tensile and flexural crack width. They were experimentally obtained and cannot be applied directly to shear crack width prediction, because

shear cracking is caused by a different mechanism.

In reinforced concrete beams subjected to shear forces, shear cracks form diagonally with an inclination towards the axis of the beam. These inclined shear cracks can begin as flexural cracks or inside the web area. According to ASCE-ACI Joint Committee 426 (ASCE-ACI 1973), the shear failure mechanism in RC beams is characterized by the occurrence of inclined shear cracks either before or after a flexural crack forms nearby. The objective of obtaining a better understanding of the shear resisting mechanism of RC beams without shear reinforcement has resulted in numerous research works (Kim and White 1991; Sato *et al.* 2004). In addition, Ueda *et al.* (1995) have presented well new truss model to explain shear resisting mechanism in RC beams with shear reinforcement. By the help of finite element program including modified constitutive models, the relationship between shear reinforcement stress and applied shear force can be well predicted under not only loading but also unloading. All these studies have enhanced our knowledge to design for shear, but the shear cracking mechanism in RC beams remains to be fully understood.

A number of investigations in last decades (Adebar and Leeuwen 1999; Adebar 2001; De Silva *et al.* 2005, 2008; Hassan *et al.* 1985, 1987, 1991; Witchukreangkrai *et al.* 2004, 2006; Piyamahant 2002; Collins *et al.* 2007; Sherwood *et al.* 2007; Zararis 2003) were focused on shear cracking mechanism and diagonal shear failure in RC members. In spite of these studies, the factors affecting the spacing between shear cracks and shear cracks width are still not known for all conditions.

Many researchers (Adebar and Leeuwen 1999; Adebar 2001) examined the effectiveness of amounts and arrangement of side-face reinforcement for flexural and

¹Ph.D. student, Division of Built Environment, Hokkaido University, Japan.

*E-Mail:*eng_zakaria@yahoo.com

²Professor, Division of Built Environment, Hokkaido University, Japan.

³Professor, Department of Civil Engineering, Dalian University of Technology, China.

⁴Ph.D. student, Department of Civil Engineering, Dalian University of Technology, China.

shear crack control in large concrete beams. It was indicated that shear crack width is generally wider than tensile crack width or flexural crack width in members with orthogonal reinforcement. The reason is due to the diagonal strain being larger than the longitudinal strain as well as the shear reinforcement being at angle to diagonal cracks (Adebar 2001). In addition, it was found that the maximum shear crack width can be affected by various parameters such as the longitudinal strain of the longitudinal reinforcement on the flexural tension side, the amount and distribution of transverse reinforcement and the side concrete cover to the transverse reinforcement (Adebar and Leeuwen 1999). In the previous study (De Silva *et al.* 2008), however, it was found that the effect of side concrete cover to the transverse reinforcement on controlling shear crack width in RC beams was not well pronounced due to the limited observation cases and studied values.

Hassan *et al.* (1985, 1987 and 1991) carried out one of the most significant studies concerning shear cracking mechanism in RC beams. In those studies, the factors to affect shear crack width which were shear reinforcement characteristics (bond characteristics, spacing, angle with member axis and its configuration) and ratio of shear span to depth were well investigated. Furthermore, Witukreangkrai *et al.* (2004, 2006) reported that the stirrup ratio has an important effect on shear crack width in RC beams and prestressed concrete beams. While Piyamahant (2002) showed that shear crack width depends on the compressive strength of concrete and diameter of the shear reinforcement.

Recent tests (Collins *et al.* 2007 and Sherwood *et al.* 2007) demonstrated that the size effect in shear is caused by the reduced ability of wide cracks to transmit shear stress. It was confirmed that the size effect is controlled by the diagonal crack spacing which is mainly influenced by the maximum distance from the longitudinal reinforcement. The spacing between shear cracks at the mid-height of the web of RC beams increases with increasing the size of the beam. This observation is similar to the findings concluded by Shioya *et al.* (1989) who conducted the most extensive series of tests to study the size effect in RC beams. It was found that the horizontal spacings between shear cracks at the mid-height of the web RC beams are about $0.5d$ over the entire range of the tested depths. Furthermore, the effectiveness of longitudinal reinforcement for shear crack control was investigated in the previous study (Zararis 2003). It was reported that the amount of the longitudinal reinforcement can have a significant effect on the opening of critical shear cracks.

Based on the previous literature survey as shown above, it can be concluded that the understanding of shear cracking behavior in RC beams has not been well clarified. This paper tries to throw the light on shear crack displacement and its mechanism in RC beams, which is a part of an extensive research between Hokkaido University in Japan and Dalian University of

Technology in P. R. China. The objectives of the present study are to clarify the effect of beam size, shear span to depth ratio, side concrete cover to stirrup, stirrup spacing, stirrup configuration, longitudinal reinforcement ratio and loading paths (loading, unloading and reloading paths) on the diagonal crack spacings and shear crack width-stirrup strain ($w-\epsilon_w$) relationship in RC beams by conducting the experiment of 10 simply supported beam specimens. In the experiment shear crack displacements measurement was conducted using demec mechanical strain gauge with a precision of 0.001 mm to measure shear crack displacements in shear cracking zone. Also, Strain gauges were mounted on the stirrups with the purpose of clarifying the relation between shear crack displacement components and stirrup strain at the intersection with shear cracks. The experimental results clearly indicating the effects of the studied parameters are valuable information for practical design codes in which the development of a rational shear crack displacement prediction method is quite needed for ensuring adequate performance for RC structures.

2. Shear and flexural cracking in reinforced concrete members

Generally, the control of cracking in concrete structures is a desirable matter to satisfy durability and serviceability requirements. In the available literature, many investigations in regard to tensile and flexural cracks for RC members were conducted during last decades (Broms 1965; Clark 1956; Gergely and Lutz 1968), but for shear cracks much concern should be given. The aim of this section is to show the main differences between shear cracking mechanism and flexural cracking mechanism in RC members.

As matter of fact, shear crack opening displacements (or width) in RC members are usually accompanied by shear crack sliding displacements (or slip) along shear cracks which create shear transferred by aggregate interlock. Shear sliding displacement (slip) which is related to shear opening displacement (width) is a main factor for fracturing of shear reinforcement, especially under cyclic loading. Conversely, in the regions of constant bending moment only tensile and flexural crack width occur without sliding along the crack.

In further studies (Hassan and Ueda 1987; Hassan *et al.* 1991), it was reported that shear crack opening displacements are not only produced by elongation of vertical leg of stirrup, but also are affected by slip of stirrup hooks and elongation of horizontal leg of stirrup which cause slip at the stirrup bent portion. However, flexural crack opening is usually produced by elongation of tension reinforcing bars only since there is no slip at their end.

Furthermore, it was observed in a previous study carried by Hassan and Ueda (1987) that diagonal crack spacings are not significantly affected by the type of shear reinforcing bars (plain or deformed). However,

many investigators reported that the spacing between flexural cracks is influenced by the type of reinforcing bars. In addition, shear crack opening displacements are affected by the angle between the shear reinforcement and shear cracks. Greater shear crack widths were found in the beams with vertical stirrups rather than with inclined stirrups at the same stirrup strain (Hassan *et al.* 1985). A shear crack generally crosses shear and tension reinforcement diagonally, while a flexural crack intersects perpendicularly main longitudinal reinforcement.

It was noticed previously (Gergely and Lutz 1968) that the flexural crack width was found to vary directly with the distance from the nearby bar. Broms (1965) stated that the widths of the primary tensile cracks close to the reinforcement were found to be considerably less than the crack widths at the surface of flexural members. It was also proved in the current study that there is a significant effect of the side concrete cover to stirrup on shear crack width at the surface of RC members. Shear crack width varies directly with the distance from the nearby stirrup due to the bond between the stirrup and the surrounding concrete. This result coincides with the concepts of flexural cracks mechanism.

Collins and Mitchell (1991) found that the diagonal crack spacing ($s_{m\theta}$) can be related to the crack control characteristics of both the longitudinal and transverse

reinforcement, which can be represented by vertical and horizontal crack spacing (s_{mx} and s_{my}), as illustrated in Fig. 1. The vertical and horizontal crack spacings are the spacings which would occur under the tension in the direction perpendicular to the longitudinal and transverse

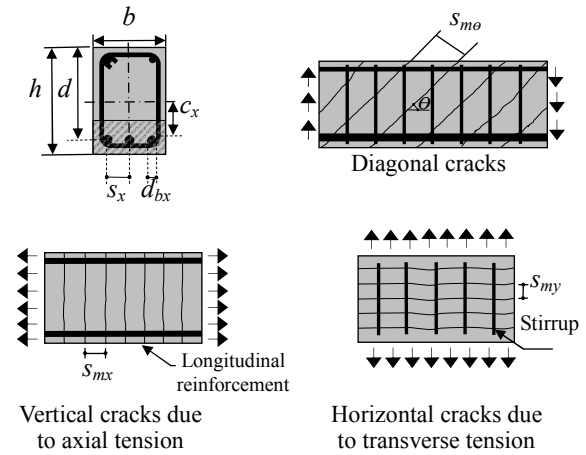


Fig. 1 Characteristics of diagonal crack spacing (Collins and Mitchell 1991).

Table 1 Details of investigated specimens.

Specimen	Overall height h , mm	Effective depth d , mm	Shear span a , mm	a/d	Side concrete cover to stirrup c_s , mm	ρ_t %	ρ_w %	Stirrup spacing s_y , mm
Series I								
A ₁ left	200	160	320	2.0	25	2.86	0.72	100
A ₁ right	200	160	480	3.0	25	2.86	0.72	100
A ₂ left	350	280	560	2.0	25	2.83	0.72	100
A ₂ right	350	280	840	3.0	25	2.83	0.72	100
A ₃ left	500	432	864	2.0	25	2.84	0.72	100
A ₃ right	500	432	1296	3.0	25	2.84	0.72	100
A ₄ left	750	669	1338	2.0	25	2.84	0.72	100
A ₄ right	750	669	1003	1.5	25	2.84	0.72	100
Series II								
B ₁ left*	500	432	864	2.0	40	2.84	0.72	100
B ₁ right	500	432	864	2.0	40	2.84	0.72	100
B ₂ left	500	432	864	2.0	60	2.84	0.36	200
B ₂ right	500	432	864	2.0	60	2.84	0.72	100
B ₃ left	500	432	864	2.0	80	2.84	0.36	200
B ₃ right	500	432	864	2.0	80	2.84	0.72	100
Series III								
C ₁	500	450	900	2.0	25	1.62	0.72	100
C ₂	500	427	854	2.0	25	2.30	0.72	100
C ₃	500	417	834	2.0	25	3.64	0.72	100

a/d : Shear span to depth ratio

ρ_t : Longitudinal reinforcement ratio

ρ_w : Shear reinforcement ratio

* Special stirrup configuration

reinforcement. Also, it was realized by Zararis (2003) that the amount of shear reinforcement probably is not the only factor to control the shear crack width. The amount of longitudinal reinforcement can have a significant effect on the opening of critical shear cracks. While tensile and flexural cracks would be affected by longitudinal reinforcements only.

On the basis of the literature review and the above discussion which summarizes the differences between shear cracking mechanism and flexural cracking mechanism in RC members, it is obvious that shear cracking is more critical and more difficult to be controlled than cracking due to axial tension or bending. Yet shear cracking mechanism and its influential parameters has not been fully understood. There is a necessity of carrying out this study.

3. Experimental program

3.1 Specimens

The experimental program consists of 10 simply supported RC beam specimens with rectangular cross section and constant breadth (b) of 200 mm. The investigated beams have been divided into three series with parameters of the beam size, shear span to depth ratio, side concrete cover to stirrup, stirrup spacing, stirrup configuration and longitudinal reinforcement ratio in order to examine their effects on shear cracking behavior in RC beams. The values of the studied parameters are given in **Table 1**.

Figure 2 shows the typical details and reinforcement arrangement of specimens in series I. It includes four unsymmetrical specimens (specimen A_1 , specimen A_2 ,

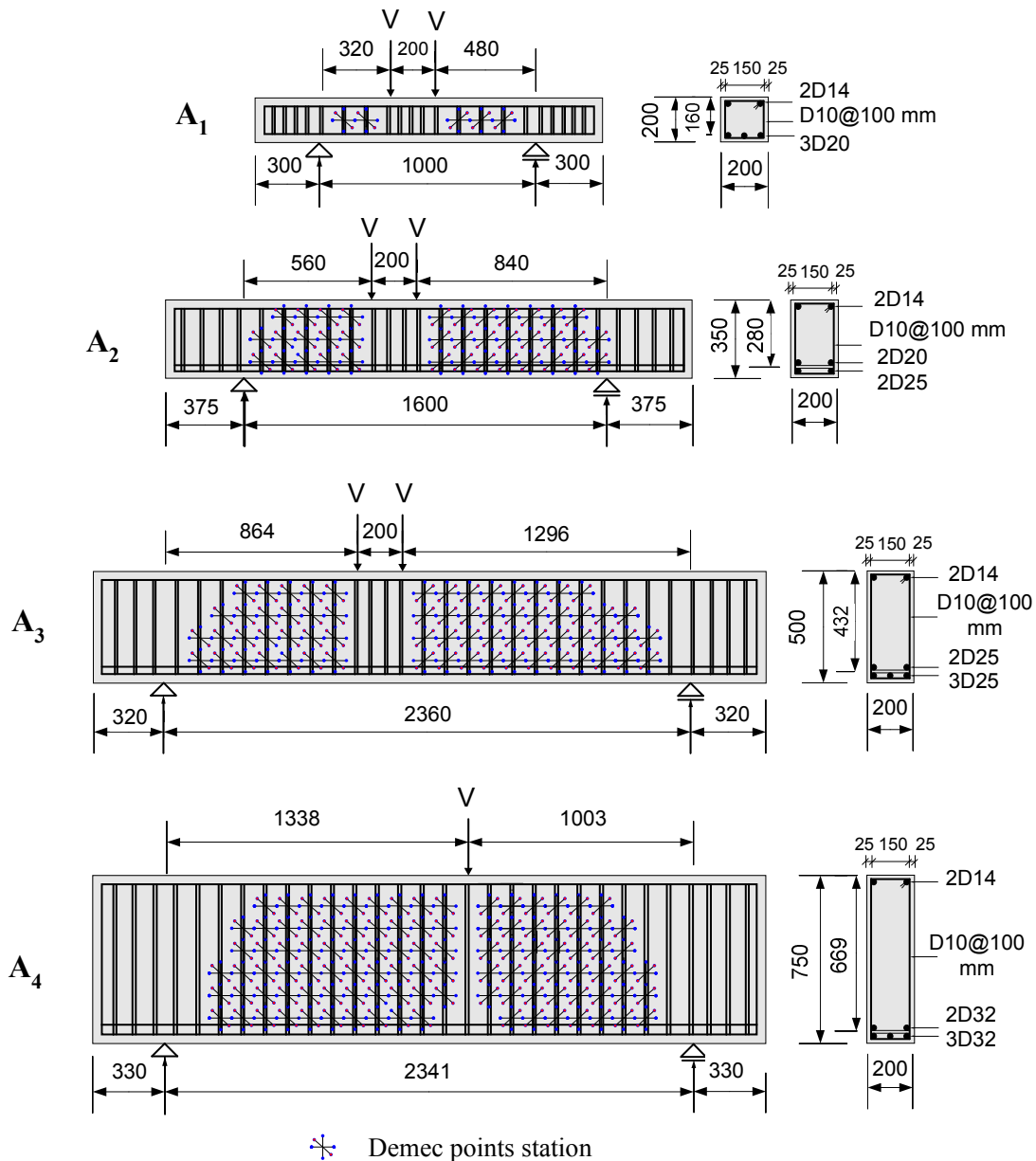


Fig. 2 Typical details and cross-sections of specimens in series I (unit: mm).

specimen A₃ and specimen A₄) whose left and right shear spans were prepared unsymmetrically to clarify the effect of beam size and shear span to depth ratio on shear crack opening displacements. The beam size in this series was varied as 200, 350, 500 and 750 mm, as given in **Table 1**. Specimen A₃ left shear span has been considered as the reference beam in regard to the effects of beam size, shear span to depth ratio, side concrete cover to stirrup and longitudinal reinforcement ratio within this study.

Layout and cross sections of specimens in series II and series III can be seen in **Fig. 3**. Side concrete cover to stirrup (*c_s*), stirrup spacing (*s_y*) as well as stirrup configuration have been deeply studied in series II which consists of three unsymmetrical specimens (specimen B₁, specimen B₂ and specimen B₃). The side concrete cover to stirrup in this series was varied as 40, 60 and 80 mm, as given in **Table 1**. Specimen B₁ has two different configurations of stirrup, one configuration for each half of the specimen, as shown in **Fig. 3**. Concerning specimens in series III, the effect of longitudinal reinforcement ratio (see **Fig. 3**) on shear crack width has been studied. This series consists of three symmetrical specimens (specimen C₁, specimen C₂ and specimen C₃) which were

designed to have longitudinal reinforcement ratio varied as given in **Table 1**.

3.2 Materials

All 10 specimens were cast in wooden molds using ready mix concrete with a characteristic strength of 40 MPa and a maximum size of aggregate of 25 mm. In the mix proportions of concrete, ordinary Portland cement was used and water-cement ratio was kept at 0.50 with the addition of an admixture (see **Table 2**).

In all test specimens, deformed reinforced bar with a diameter of 10 mm was used for the shear reinforcement. Also, in all the specimens, the shear reinforcement was used as a closed (rectangular) stirrup configuration, except in specimen B₁ left shear span with a special stirrup configuration, as illustrated in **Fig. 3**. The mechanical properties of reinforcement used are given in **Table 3**.

3.3 Instrumentation and test procedure

Shear crack investigation was conducted using demec digital mechanical strain gauge with a precision of 0.001 mm (see **Fig. 4**), to measure concrete deformations in shear cracking zone. Before carrying out the test, contact

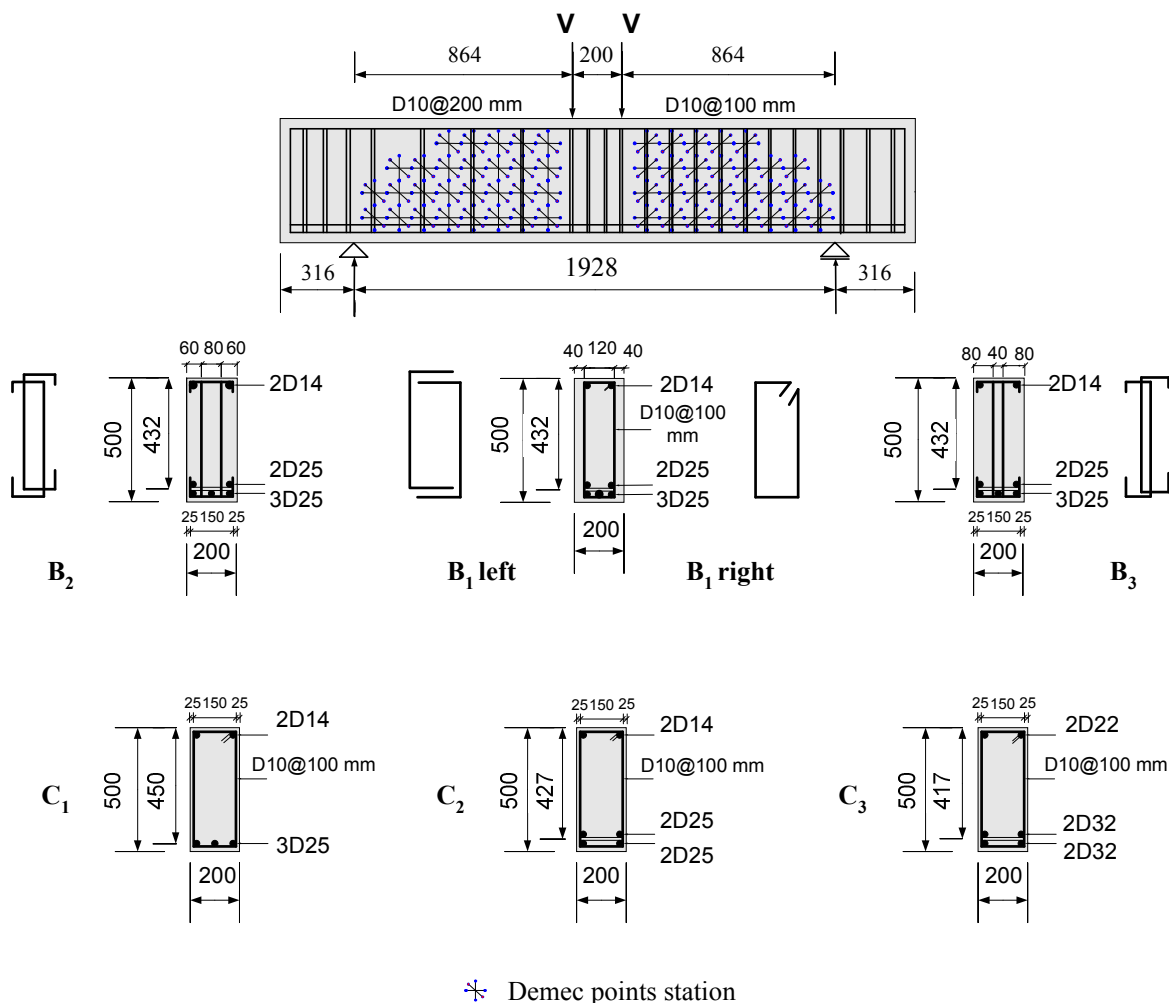


Fig. 3 Typical details and cross-sections of specimens in series II and series III (unit: mm).

Table 2 Mix proportions of concrete.

Characteristic cube strength	40 MPa
Cement type	Ordinary Portland cement
Maximum aggregate size	25 mm
Slump for concrete	180 mm
Free-water content	180 kg/m ³
Cement content	360 kg/m ³
Coarse aggregate content	1040 kg/m ³
Fine aggregate content	723 kg/m ³
Fly ash	69 kg/m ³
Water-cement ratio	0.50
Admixture	9 kg/m ³

Table 3 Mechanical properties of reinforcement.

Bar diameter (mm)	Area (mm ²)	Yield strength (MPa)	Elastic modulus X10 ³ (MPa)	Ultimate strength (MPa)
D10	72.41	370	173	580
D14	141.08	435	182	666
D20	305	425	187	656
D22	353.13	450	198	639
D25	490.87	450	200	649
D32	759.95	465	200	618

chips designate as demec points which are points for contact rosette strain gauges were attached to the concrete surface with adhesive. Typical locations for rosette demec points are shown in **Fig. 2** as well as **Fig. 3**. Generally, the rosette demec point stations can be used to measure the concrete strain in the horizontal X, vertical Y and Diagonal Z using 100 mm gauge length, as shown in **Fig. 5**. The measurements of concrete strains combined with the record of shear crack angle can be used to estimate the two components of shear crack displacements along shear crack (Hassan *et al.* 1985); one is defined as shear crack opening (width) in the direction perpendicular to shear crack, and the other is shear crack sliding (slip) that occurs in the direction of shear crack, as shown in **Fig. 5**. Only the demec point stations intersecting with single shear crack were investigated. The shear crack angle corresponds to each shear crack intersecting with a certain demec point station was measured as the average angle of the shear crack at that station. In this sense, the measured crack width is an average in the demec point station whose size is 100 mm. This averaging way is appropriate to quantify shear crack opening displacement which is necessary for design purpose.

To clarify the relation between shear crack displacements components and stirrup strain at the intersection with shear cracks, electrical strain gauges were also attached to all the studied stirrups notated by numbers at given locations, as shown in **Figs. 6, 7** and **8**. Furthermore, two strain gauges were installed on the main reinforcement below each applied loading point. The examined surface of the test beams was white-painted to



Fig. 4 Monitoring of shear crack displacements by using demec points strain gauge.

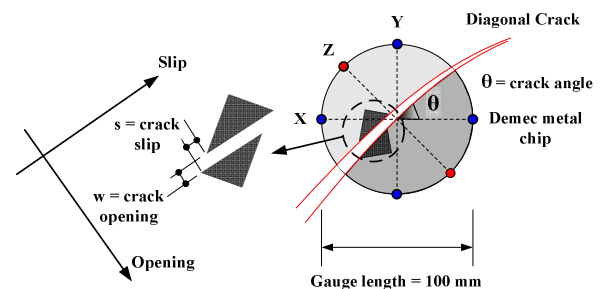


Fig. 5 Definition of shear crack displacements within the rosette demec stations.

help in observation of the cracks propagation during the experiment. All the specimens were tested under monotonic point loading using a 5000 kN testing machine. Collection of data was done by using data logger system. Load was halted at several stages during the test to mark the cracks and take photographs. Also, detailed data consisting of horizontal, vertical and diagonal deformations of concrete as well as strain gauges readings was taken at each load stage.

4. Experimental results and discussion

4.1 Crack patterns and spacing

Crack patterns of all the tested specimens at failure load together with shear crack angles are shown in **Figs. 6, 7** and **8**. The shear crack angles were measured as an average angle of each major crack. As seen from the figures, it can be concluded that the shear crack angles vary along the shear span. The shear cracks near the loading point show the steeper shear crack angle. By moving to the supporting point these angles start to decrease. The first batch of cracks was flexural cracks occurring in the mid-span zone, and as the load increased, a series of flexural cracks was formed in the shear span region. Then, it rotated to form flexural-shear cracks joining the loading and supporting points, and additional shear cracks appeared during the subsequent loading stages. All the tested specimens failed in shear failure mode, except specimen C₁, in which flexural yielding occurred

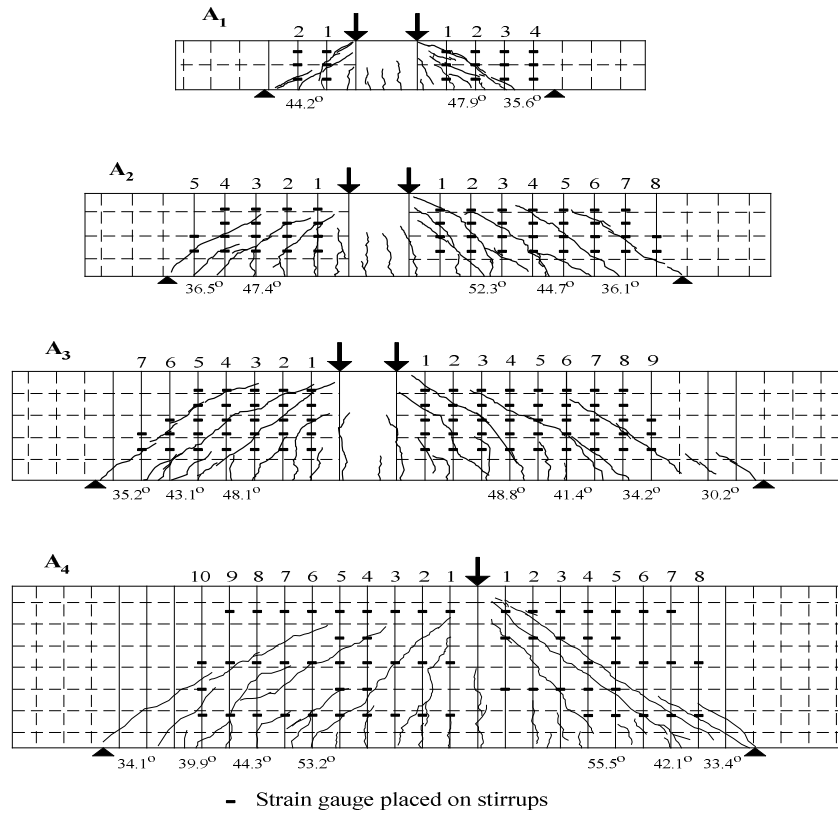


Fig. 6 Typical strain gauge locations and crack patterns of specimens in series I.

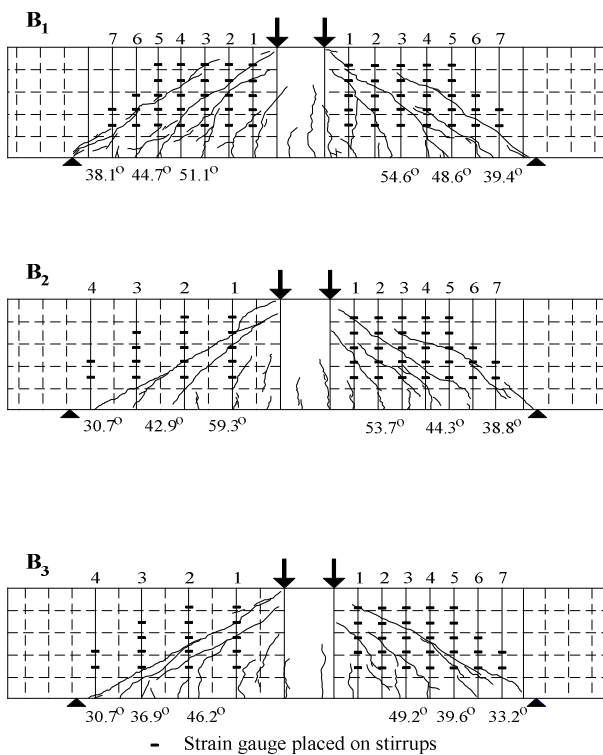


Fig. 7 Typical strain gauge locations and crack patterns of specimens in series II.

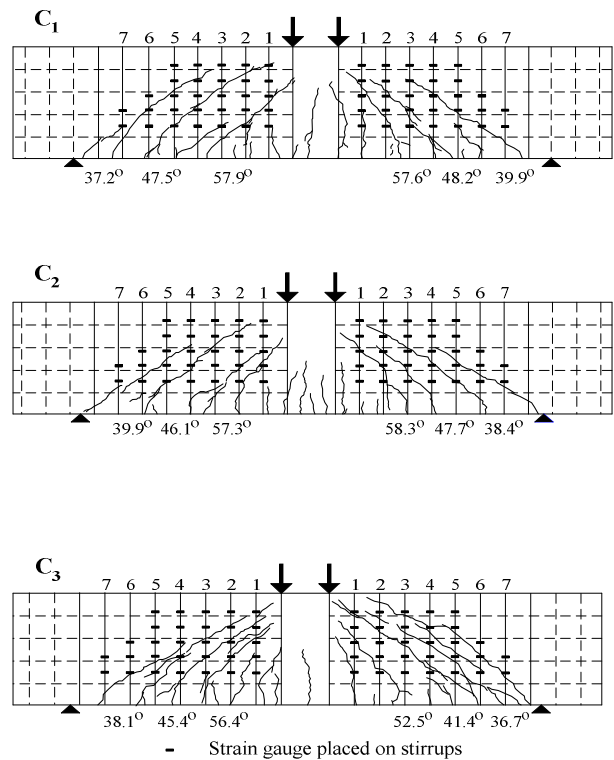


Fig. 8 Typical strain gauge locations and crack patterns of specimens in series III.

first followed by yielding of stirrups. Shear failure mode resulted in significant and wider shear cracks in the shear

cracking zone.

Generally, there are two phases after the cracking,

starting from the first crack up to the failure; one is the crack formation phase in which new shear cracks occur, and the other is the stabilized cracking phase in which only shear cracks widening is supposed to occur. The results presented in this section and the study conducted by Adebar (2001) indicate that with increasing the shear stresses in RC cracked members, the concrete between shear cracks deforms significantly resulting in decreasing the diagonal crack spacing.

It is well known that the crack opening displacement at a shear crack-stirrup intersection increases in general with increasing shear reinforcement strain and with increasing the spacing between shear cracks. Thus, it is more reasonable to investigate the diagonal crack spacings in the current experiment. The diagonal crack spacing ($s_{m\theta}$) is considered to be a function of the ability of the reinforcement in the longitudinal and transverse directions to control the vertical crack spacing (s_{mx}) and horizontal crack spacing (s_{my}), respectively (CEB-FIP 1990, Collins and Mitchell 1991).

The measured values of shear crack angles for each major crack and its average (θ_{avg}), in addition to the measured values of the diagonal crack spacings along the shear span of each specimen are given in **Table 4**. The values of the diagonal crack spacings were measured at the intersection of main shear cracks with the centroid of beam section in the direction perpendicular to the shear crack, as shown in **Fig. 9** indicating the case of beam A₄ left shear span. Also, measured flexural cracking load, shear cracking load, ultimate failure load and the predicted shear cracking and ultimate strength based on

JSCE design code (JSCE 2002) for each investigated specimen are given in **Table 4**.

The effects of beam size, shear span to depth ratio, side concrete cover to stirrup, stirrup spacing, stirrup configuration and longitudinal reinforcement ratio on the spacings between shear cracks are explained in the following discussion.

(1) Size effect

By examining the shear spans with the same conditions except for the size (or beam height) of the tested specimens in series I, it was observed that the diagonal crack spacing increases with increasing the size of the beam. The measured value of diagonal crack spacing for specimen A₄ left shear span is greater than the measured ones for other specimens in series I, as shown in **Table 4**. The inability of the larger beams to adequately control the spacings between shear cracks is due to the reduced ability of the longitudinal reinforcement to control the vertical crack spacing (s_{mx}) at the mid-height of the larger beam. The evidence for this general observation is discussed in more details.

The average vertical crack spacings of shear cracks at $0.5d$, $0.75d$ and at d (see **Fig. 9**) from the top of specimens A₄ left shear span, A₃ left shear span and A₂ left shear span are plotted in **Figs. 10 (a)** through **(c)** versus the shear stress. In these figures, it can be seen that there is little difference in the crack spacing at the level of the main longitudinal reinforcement among the investigated specimens (large and small specimens). Also, it is interesting to notice that in the small specimen (A₂ left shear

Table 4 Measured diagonal crack spacing, shear crack angles, cracking loads and failure loads.

Specimen	Measured diagonal crack spacing, mm	Measured shear crack angles, degree					$V_{f,cr}$ (kN)	V_c (kN)	$V_{c,JSCE}$ (kN)	V_u (kN)	$V_{u,JSCE}$ (kN)
		θ_1	θ_2	θ_3	θ_4	θ_{avg}					
Series I											
A ₁ left	47.3	44.2	-	-	-	44.2	60.0	120.0	72.0	285.4	234.1
A ₁ right	69.9	47.9	35.6	-	-	41.8	60.0	160.0	103.7	-	234.1
A ₂ left	105.7	47.4	36.5	-	-	42.0	80.0	140.0	126.2	470.2	409.9
A ₂ right	120.6	52.3	44.7	36.1	-	44.4	100.0	180.0	181.7	-	409.9
A ₃ left	143.3	48.1	43.1	35.2	-	42.1	150.0	200.0	175.0	720.0	609.4
A ₃ right	158.7	48.8	41.4	34.2	30.2	38.7	160.0	280.0	258.1	-	609.4
A ₄ left	191.2	53.2	44.3	39.9	34.1	42.9	280.0	380.0	334.8	1196.5	1063.1
Series II											
B ₁ left	152.3	51.5	44.7	38.1	-	44.8	160.0	200.0	206.5	715.0	609.4
B ₁ right	154.3	54.6	48.6	39.4	-	47.5	160.0	220.0	206.5	-	609.4
B ₂ left	193.2	59.3	42.9	30.7	-	44.3	160.0	280.0	206.5	540.7	407.9
B ₂ right	171.8	53.7	44.3	38.8	-	45.6	160.0	280.0	206.5	-	609.4
B ₃ left	215.2	46.2	36.9	30.7	-	37.9	200.0	300.0	206.5	522.4	407.9
B ₃ right	195.6	49.2	39.6	33.2	-	40.7	200.0	300.0	206.5	-	609.4
Series III											
C ₁	157.2	57.6	48.2	39.9	-	48.6	160.0	180.0	177.1	551.4	596.5
C ₂	146.9	58.3	47.4	38.4	-	48.0	160.0	240.0	190.7	600.8	588.7
C ₃	128.1	52.5	41.4	36.7	-	43.5	200.0	280.0	218.5	760.3	607.1

$V_{f,cr}$: Measured flexural cracking load V_c : Measured shear cracking load $V_{c,JSCE}$: Predicted shear cracking strength (JSCE 2002)
 V_u : Measured ultimate failure load $V_{u,JSCE}$: Predicted ultimate shear strength (JSCE 2002)

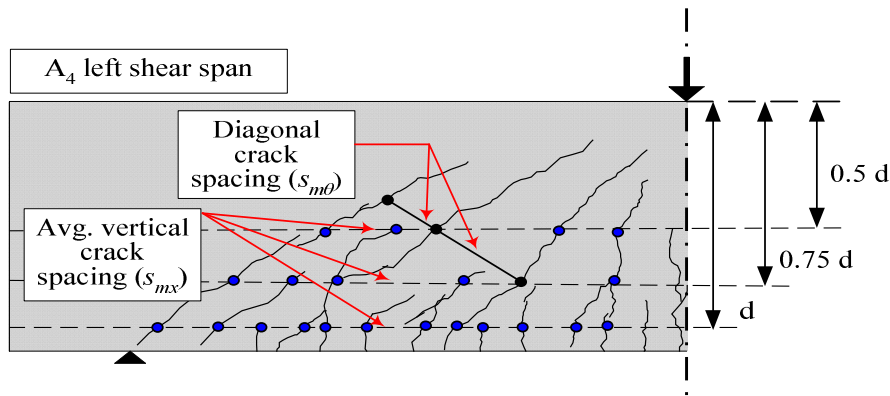


Fig. 9 Diagonal crack spacings for specimen A₄ left shear span.

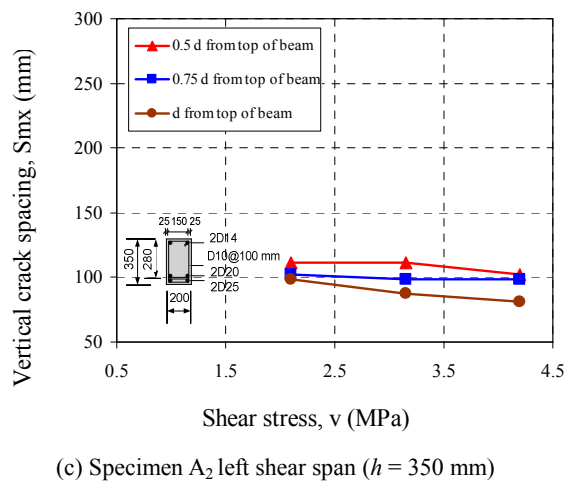
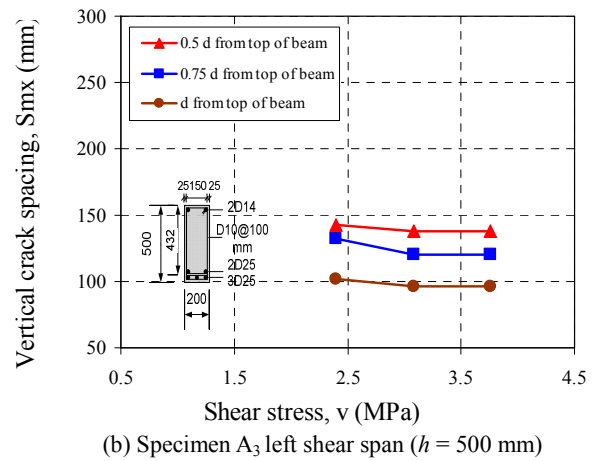
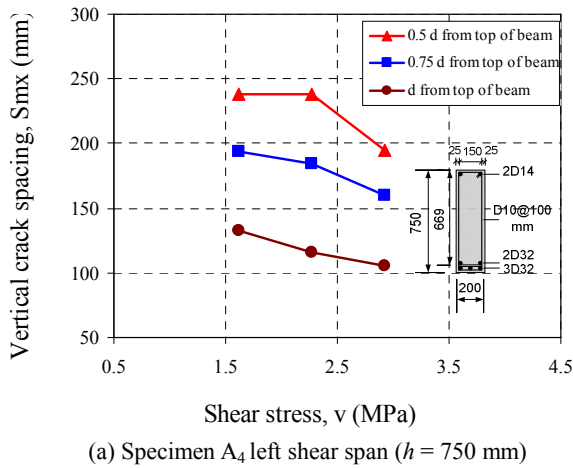


Fig. 10 Vertical crack spacings in specimens A₄ left shear span, A₃ left shear span and A₂ left shear span.

span) there is very little difference in the crack spacings at different depths of the beam at the same shear stress, while considerable increase in the crack spacings in the large specimen (A₄ left shear span) can also be seen.

The obtained results agree with those in the previous studies (Collins *et al.* 2007, Collins and Mitchell 1991, Sherwood *et al.* 2007 and Shioya *et al.* 1989). On the other hand, it was reported in previous studies (Adebar and Leeuwen 1999; Collins *et al.* 2007) that using additional layers of longitudinal steel distributed along the side faces of large concrete members resulted in adequate control of the spacings between shear cracks. The reason such that additional longitudinal reinforcement, which is provided at mid-height of concrete sections, may actively reduce the spacings is due to the increasing bond effect between the longitudinal steel and the surrounding concrete. However, a further investigation is needed to clarify the effect of using additional longitudinal steel along the side faces of large concrete members and its arrangements on controlling the spacings between shear cracks in large RC members.

(2) Effect of shear span to depth ratio

The average vertical spacings of shear cracks at 0.5d and d from the top of specimens A₂ and A₃ are plotted in Figs. 11 (a) and (b) versus the shear stress. Both figures show the variation of the average vertical crack spacings of specimens with the identical condition except the shear span to depth ratio (a/d).

The experimental results reveal that the spacing between shear cracks for the specimens with larger shear span to depth ratio ($a/d = 3.0$) is greater than the spacing between shear cracks in the case of specimens with smaller shear span to depth ratio ($a/d = 2.0$), as shown in **Table 4** as well as **Figs. 11 (a)** and **(b)**. Possible reason for this behavior is due to the difference in flexural crack spacings (crack spacings at d from the top of specimens) between the investigated shear spans for each specimen. It can be noticed that the flexural crack spacing in the specimens with $a/d = 2$ is smaller than the flexural crack spacing in the specimens with $a/d = 3$, as shown in **Figs. 11 (a)** and **(b)**. The higher local bond stress along flexural reinforcement due to shorter shear span, which induces greater moment change per unit length, is considered to be the reason of the smaller spacing. The obtained result coincides with the ones in the previous study (Hassan *et al.* 1985).

Since the current experiment has limited cases, the additional investigation is considered to be necessary to disclose the effect of shear span to depth ratio on the spacing between shear cracks.

(3) Effect of side concrete cover to stirrup

Table 4 shows the diagonal crack spacings of specimens in series II. Side concrete cover to stirrup (c_s) in this series was designed to be 40, 60 and 80 mm for specimens B₁, B₂ and B₃, respectively. It can be observed that the diagonal crack spacings for specimen B₃ right shear span ($c_s = 80$ mm) and specimen B₂ right shear span ($c_s = 60$ mm) are greater than diagonal crack spacing for specimen B₁ right shear span ($c_s = 40$ mm), implying that side concrete cover to stirrup (c_s) has a significant effect on diagonal crack spacing. In general, increasing the side concrete cover to stirrup leads to larger diagonal crack spacings. The increase in diagonal crack spacing in case of greater side concrete cover to stirrup is because of the reduced ability of stirrups to control crack spacings at the surface of RC members.

It is well known that slip of the stirrup through the concrete induces bond stresses which transfer force from the stirrup to the concrete between cracks. In order to form a crack within an effective concrete area ($A_{c,ef}$) around the stirrup, the force which has to be introduced to concrete by bond at the end of the transfer length should exceed the cracking strength of concrete. Increasing the side concrete cover to stirrup increases the effective concrete area in which the crack width should be well controlled. Then, the force which has to be introduced into the surrounding concrete by bond should increase to cause cracking on the surface of RC members. Hence, larger transfer length (or crack spacing) is necessary.

(4) Effect of stirrup spacing (or stirrup ratio)

The measured values of diagonal crack spacing of specimens B₂ and B₃ are given in **Table 4**, each of which includes the comparison of two cases with the same

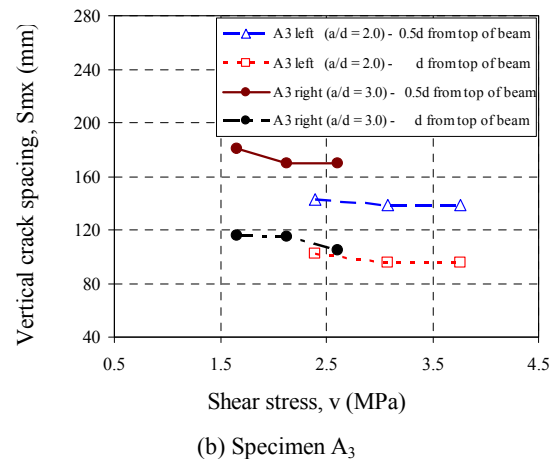
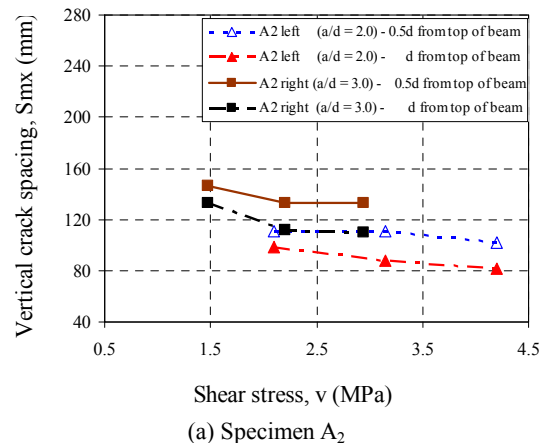


Fig. 11 Effect of shear span to depth ratio on the spacings between shear cracks.

condition except for stirrup spacing (or stirrup ratio). In each specimen, the stirrup spacing was designed to be 100 and 200 mm for the right and left shear span, respectively. It can be inferred that the greater stirrup spacing (s_y) leads to the greater diagonal crack spacing, confirming that there is a significant influence of the stirrup spacing on the spacing between shear cracks.

The reason for this behavior is the decreasing effective concrete area, in which shear crack width is controlled by the stirrup, and hence the increasing bond effect between the stirrups and the surrounding concrete. Increasing the bond effect results in reducing the transfer length (or crack spacing) in which the forces to cause a crack are transferred into the concrete between cracks by the bond stresses.

Another possible reason for this behavior is due to the difference in flexural crack spacing as shown in **Fig. 7**. Stirrups often act as crack initiators and thus affect the flexural crack spacing as proved by Rizkalla *et al.* (1983). Hence, smaller flexural crack spacing would be formed in the case of closer stirrup spacing.

(5) Effect of stirrup configuration

By investigating the crack patterns of specimen B₁ shown in Fig. 7 and measured diagonal crack spacing given in Table 4, it can be noticed that there is no significant difference in diagonal crack spacing between shear spans with closed stirrup configuration and with open stirrup configuration. Number and inclination of shear cracks are seemingly the same between them. The crack patterns and spacings seem not to be changed among investigated stirrup configuration.

(6) Effect of longitudinal reinforcement ratio

The measured values of diagonal crack spacing and the crack patterns of specimens in series III are given in Table 4 and Fig. 8. The longitudinal reinforcement ratio (ρ_l %) is varied as 1.62%, 2.30% and 3.64% for specimens C₁, C₂ and C₃, respectively. It can be inferred that the higher longitudinal reinforcement ratio brings the smaller diagonal crack spacing.

The reason such that the larger amount of longitudinal reinforcement can actively reduce the spacings between shear cracks is due to increasing the bond effect between the longitudinal reinforcement and the surrounding concrete, thereby enhancing crack control characteristics. Increasing the bond effect results in reducing the transfer length (or crack spacing) in which the forces to cause a crack are transferred into the concrete between cracks by the bond stresses.

4.2 Shear crack width-stirrup strain ($w-\varepsilon_w$) relationship

It is well known that the strain of shear reinforcement is the most important factor affecting the opening of shear cracks. With increasing the shear stresses after shear cracking, the stirrup starts to elongate causing increase in shear crack width in the surrounding concrete. For this reason, the variation of measured shear cracks width was plotted in Figs. 12 to 21 against the stirrup strain obtained from the nearest strain gauge to each particular crack location under each loading step and at the same time of measuring both components. The shear crack width variation was observed at various locations to have a reasonable understanding of shear cracking behavior at different locations.

The experimental results are shown in shear crack width-stirrup strain form, and a regression analysis representing a linear relationship between the average values of shear crack width and the stirrup strain is applied to each obtained data set for the tested specimens. It can be noticed from the experimental results presented in this section that there is a significant variation of shear crack width at a given stirrup strain. The reason for this variation is discussed in section 4.3.

Shear crack width-stirrup strain form is used as a unified basis of comparison to examine the effect of each studied parameter on the relationship between shear crack width and stirrup strain ($w-\varepsilon_w$), and for the other hand to support the development of future design method

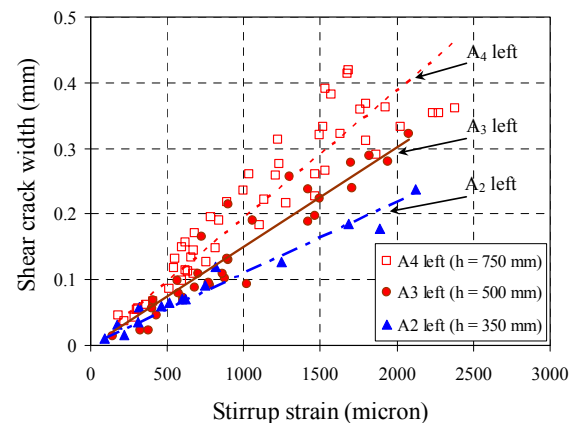


Fig. 12 Effect of beam size on shear crack width-stirrup strain relationship.

to predict shear crack displacements.

(1) Size effect

From the inclination of trend lines representing the effect of beam size on shear crack width-stirrup strain relationship as shown in Fig. 12, it can be inferred that among the shear spans with the same condition except for the size (or beam height) of the tested specimens in series I, shear crack width of the larger beam (A₄ left shear span) developed more quickly compared to the smaller beam (A₂ left shear span). It can be seen that at the same stirrup strain, shear crack width increases with increasing the size of the beam, confirming the reduced crack control characteristics of the larger beams. The reason for this behavior is considered that the larger beams cause greater diagonal crack spacings, as shown in section 4.1 (1).

Since in specimen A₁, multiple shear cracks intersected with demec point rosette stations as seen in Fig. 6, the data in specimen A₁ was excluded from the analysis for the size effect but will be used for further investigation on shear crack width.

(2) Effect of shear span to depth ratio

The shear crack width-stirrup strain relationship of specimens with the same condition but a/d ratio equal to 2.0 and 3.0 are given in Fig. 13 (a) and Fig. 13 (b). Generally, the higher is the a/d the faster is the rate of diagonal crack growth. Both figures show that greater shear crack widths were obtained in the beams with larger shear span to depth ratio (a/d), because of the observed spacing between shear cracks for shear spans with a larger a/d ratio was generally greater compared to shear spans with a smaller a/d ratio (see section 4.1 (2)). This observation shows an agreement with the previous findings obtained by Hassan *et al.* (1985).

(3) Effect of side concrete cover to stirrup

Specimens A₃, B₁, B₂ and B₃ were designed to have side

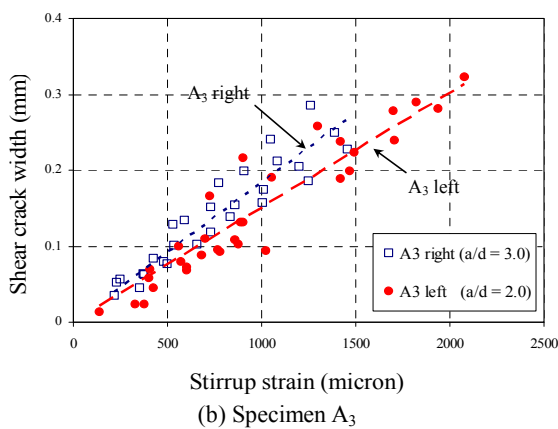
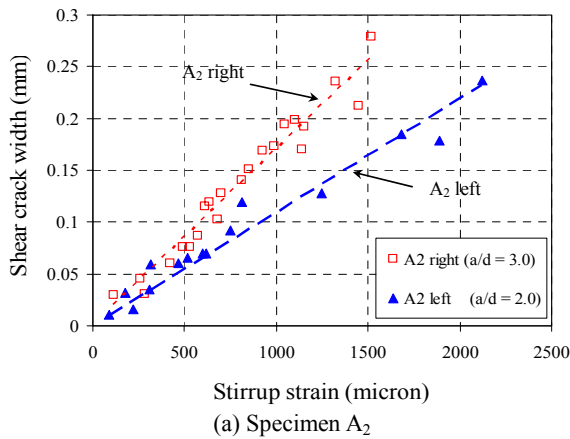


Fig. 13 Effect of shear span to depth ratio on shear crack width-stirrups strain relationship.

concrete cover to stirrup 25, 40, 60 and 80 mm, respectively. **Figure 14 (a)** shows the relationship between shear crack width and stirrup strain for specimens B_1 right shear span, B_2 right shear span and B_3 right shear span in series II in addition to specimen A_3 left shear span as the reference beam, all of which were with the identical condition except for the side concrete cover. Also, **Figure 14 (b)** shows the relationship between shear crack width and stirrup strain for specimens B_2 left shear span and B_3 left shear span, both of which were with the same condition except for the side concrete cover. From the figures, it can be seen clearly that at the same stirrup strain value, the larger side concrete cover to stirrup causes the greater shear crack width.

The main reason causing the increase in crack opening displacements in case of larger side concrete cover to stirrup is the increase in diagonal crack spacing as shown in section 4.1 (3). Additional reason for this increase in shear crack opening displacement can be explained according to the fact that the shear crack width at concrete surface increases directly with concrete cover that is the distance from the nearby stirrup. On other hand, the shear crack width at the location of stirrup is relatively smaller

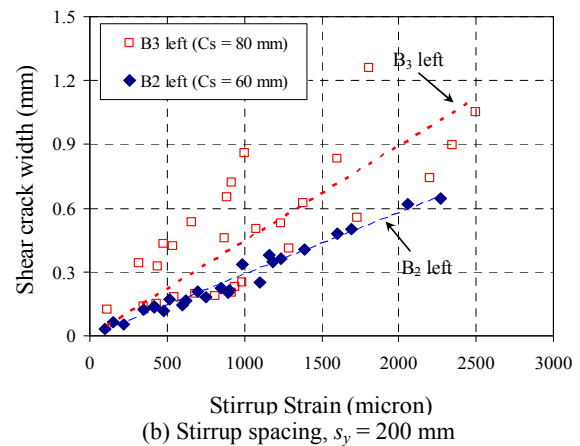
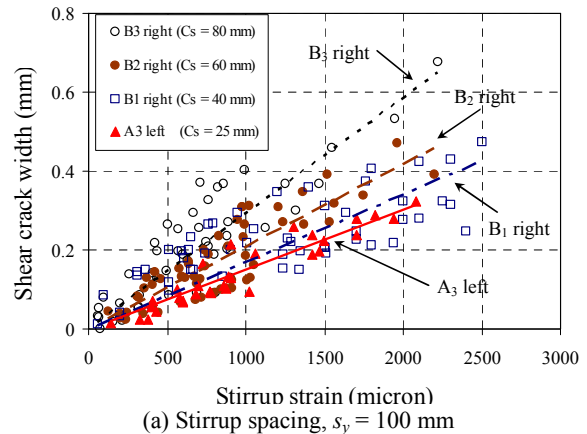


Fig. 14 Effect of side concrete cover to stirrup on shear crack width-stirrups strain relationship.

than the shear crack width at the surface of RC member due to crack width control provided by the bond between the stirrup and the surrounding concrete. To show the evidence of the additional reason for the increase in shear crack opening displacement with increasing the side concrete cover to stirrup, the shear crack width-stirrups strain data given in **Figs. 14 (a)** and **(b)** is replotted in **Figs. 15 (a)** and **(b)**. In these figures, the ratios of crack width to crack spacing are plotted versus the stirrup strain. It is interesting to notice that there is considerable increase in the ratio of crack width to crack spacing in the shear spans with larger side concrete cover to stirrup compared to shear spans with smaller side concrete cover to stirrup.

The obtained result agrees with the findings of the previous study (Adebar and Leeuwen 1999) that the shear crack width increases with increasing the side concrete cover to stirrup.

(4) Effect of stirrup spacing (or stirrup ratio)

The effect of stirrup spacing on shear crack width-stirrups strain relationship can be shown in **Figs. 16 (a)** and **(b)**, for specimens in series II, each of which shows the

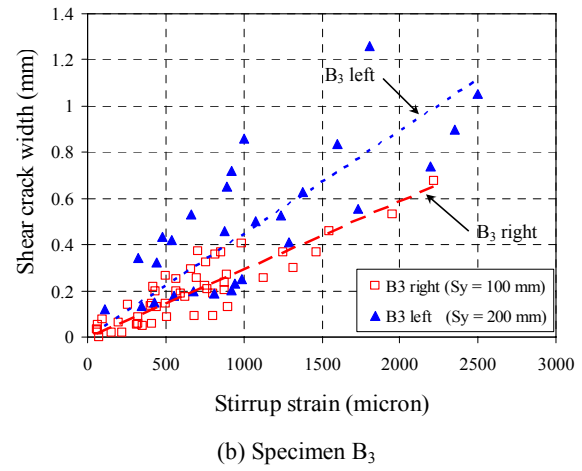
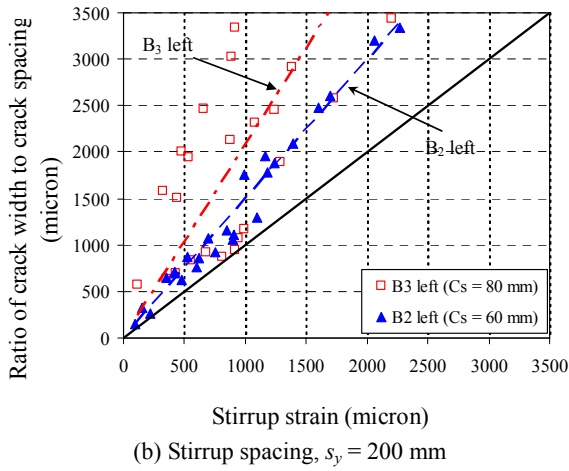
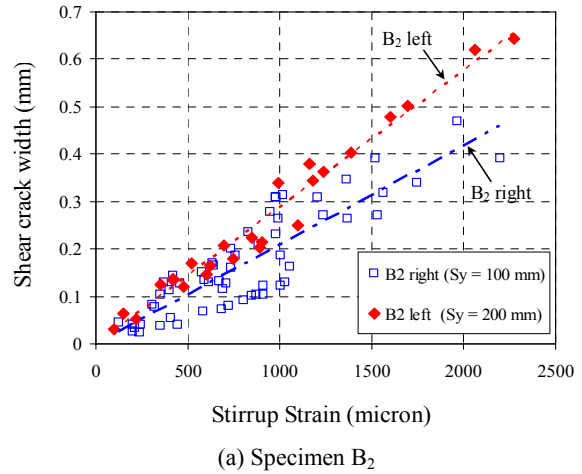
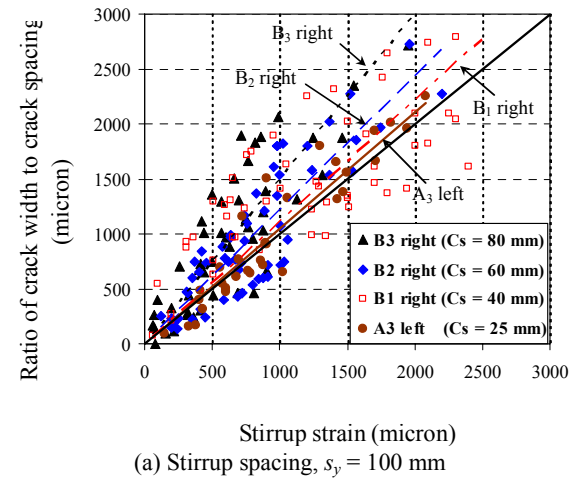


Fig. 15 Effect of side concrete cover to stirrup on ratio of crack width to crack spacing-stirrup strain relationship.

Fig. 16 Effect of stirrup spacing on shear crack width-stirrup strain relationship.

comparison of two cases with the identical condition except for stirrup spacing. Changing the stirrup spacing from 100 mm to 200 mm resulted in varying the shear reinforcement ratio from 0.72 % to 0.36 %. Both figures clarify that the relationship between shear crack opening displacement and stirrup strain seems to be changed by the stirrup spacing. Larger stirrup spacing (smaller stirrup ratio) yields greater shear crack opening at the same stirrup strain.

The primary reason causing difference in crack opening displacements is considered to be the diagonal crack spacing as discussed in section 4.1 (4). Another reason for such behavior is that the concrete area, in which shear crack width is controlled by the stirrups, increases with increasing the stirrup spacing resulting in less adequate control of shear crack width at the same stirrup strain value. It is worthy to mention that the shear crack opening at the location of stirrup is relatively smaller than the shear crack opening at locations other than the stirrup location, because of crack opening control provided by the bond between the stirrup and the surrounding concrete. Hence, wider shear crack opening

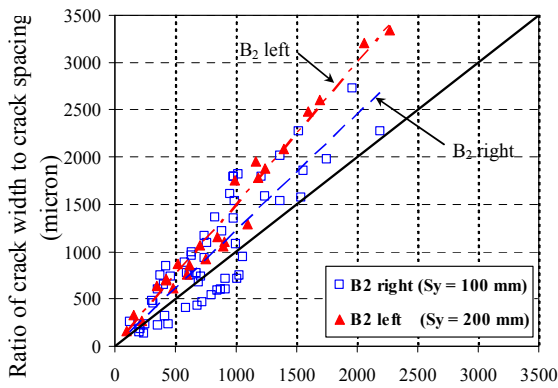
could be obtained due to increasing the stirrup spacing. The ratio of crack width to crack spacing of specimens B₂ and B₃ are plotted in **Figs. 17 (a) and (b)** versus the stirrup strain as the evidence for the influence of stirrup spacing. Both figures clearly show that the larger stirrup spacing the greater ratio of crack width to crack spacing is. Since the equality line is not observed, it can be revealed that the stirrup spacing, as the additional reason, affects significantly the shear crack width.

The effective concrete area around the reinforcing bar, in which crack width is controlled by the reinforcement, has been defined by CEB-FIP Model Code (1990) to be within $7.5 d_b$ (where d_b is the diameter of stirrup) from the center of the bar. Since the stirrup diameter is 10 mm, in the case of 200 mm stirrup spacing, the concrete area around the stirrup is larger than the effective concrete area given by CEB-FIP Model (1990); hence less adequate control of shear crack width could be achieved. Conversely, the other case of 100 mm stirrup spacing shows the smaller concrete area than the effective concrete area and that will result in more control of the width of shear cracks.

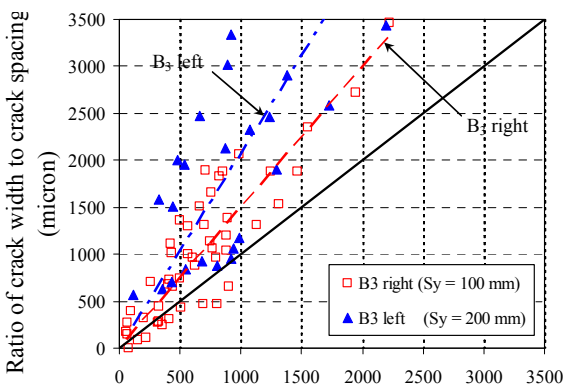
The observed effects of stirrup spacing on shear crack width-stirrup strain relationship agree with those in the previous studies (Adebar and Leeuwen 1999; Hassan *et al.* 1991; Witchukreangkrai 2006).

(5) Effect of stirrup configuration

As shown in Fig. 2, specimen B₁ has two different stirrup configuration, open stirrup configuration for the left shear span and closed stirrup configuration for the right shear span. Figure 18 clarifies the effect of stirrup configuration on shear crack width-stirrup strain relationship. The closed stirrup configuration shows smaller shear crack width in comparison to the open stirrup configuration at the same stirrup strains. All of that can be attributed to the elongation of horizontal leg of stirrups and the slip of stirrup end which affect the slip at the end of vertical leg of stirrup, that is bent portion, and hence affect the crack width as proved by Hassan and Ueda (1987). The open stirrup configuration is with less efficient hook which is straight hook or 90 degree hook than that of the closed stirrup configuration, which is 135 degree hook, as shown in Fig. 2. Hence, larger slip between stirrup and concrete occurs, causing larger shear crack width at the same stirrup strain.



(a) Specimen B₂



(b) Specimen B₃

Fig. 17 Effect of stirrup spacing on ratio of crack width to crack spacing-stirrup strain relationship.

(6) Effect of longitudinal reinforcement ratio

The influence of longitudinal reinforcement ratio (ρ_l %) on the shear crack width has been studied in series III, in addition to specimen A₃ left as the reference beam. The longitudinal reinforcement ratio (ρ_l %) is varied as 1.62%, 2.30%, 2.84% and 3.64%. Figure 19 shows clearly that the higher longitudinal reinforcement ratio yields the smaller shear crack width at the same stirrup strain. The reason is that the larger amount of longitudinal reinforcement the smaller diagonal crack spacing is (see section 4.1 (6)); hence it results in smaller shear crack width. Another possible reason for this behavior is that larger amounts of longitudinal reinforcement effectively restrict the widening of flexural cracks and their development into flexure-shear cracks. The evidence for this additional reason which shows the effect of longitudinal reinforcement ratio on the relationship between the ratio of crack width to crack spacing and stirrup strain can be seen in Fig. 20. It can be observed that the smaller is longitudinal reinforcement ratio the higher ratio of crack width to crack spacing could be obtained at the same stirrup strain, showing the reduced ability of the smaller amount of longitudinal reinforcement to control the widening of flexure-shear cracks.

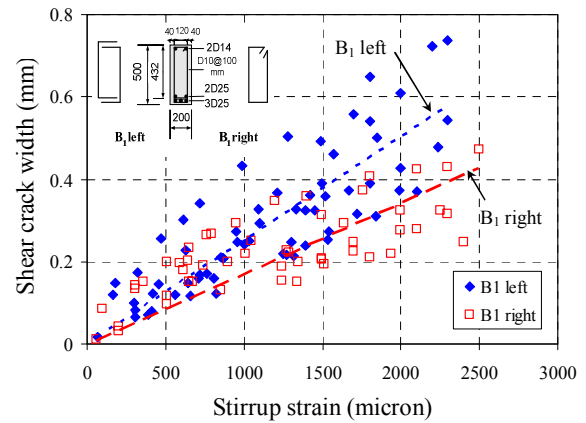


Fig. 18 Effect of stirrup configuration on shear crack width-stirrup strain relationship.

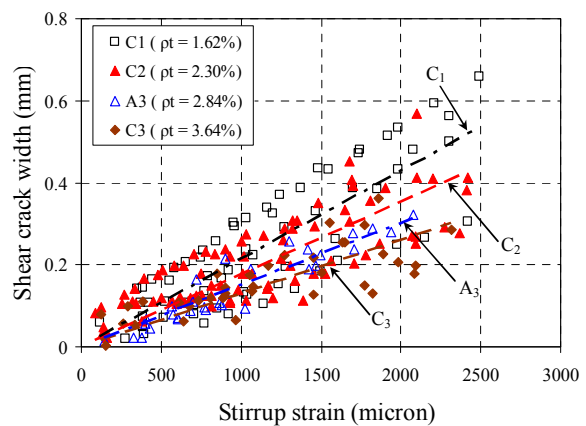


Fig. 19 Effect of longitudinal reinforcement ratio on shear crack width-stirrup strain relationship.

This result agrees with the previous studies (Angelakos 1999, Zararis 2003), who reported that the amount of shear reinforcement probably not be the only factor to control shear crack width but also the amount of longitudinal reinforcement.

(7) Effect of loading paths (loading, unloading and reloading paths)

Figures 21 (a) through (c) show the comparison of shear crack width-stirrup strain ($w-\epsilon_w$) relationship among cases of loading, unloading and reloading paths for specimens in series I. The loading test was conducted and measurements for shear cracks displacements were continued till yielding happened in either the main reinforcement steel or shear reinforcement steel in one of the shear spans in specimen. The unloading stage then was started and measurements were taken at some load intervals, till the load became null. Then, the reloading stage was continued for the other span without yielding. It can be concluded from Figs. 21 (a) through (c) that there is no significant effect on shear crack width-stirrup strain ($w-\epsilon_w$) relationship.

4.3 Variation of shear crack width along shear crack

Shear crack opening displacement (or width) varies along shear crack. An example of the variation can be seen in specimens B₂ left shear span and B₃ right shear span. The measured locations for shear crack width along main shear cracks for specimens B₂ left shear span and B₃ right shear span are given in Figs. 22 and 23, respectively. For specimen B₂ left shear span, location B is at the intersection of main shear crack with the shear reinforcement, while the other locations (locations A and C) are at a certain distance from shear crack-stirrup intersection. The distances from the shear crack tip to locations A, B and C are 21, 33 and 45 cm, respectively. Location D is at the intersection of main shear crack with the longitudinal reinforcement and shear reinforcement, while location E

is at a certain distance from the intersection of shear crack with longitudinal reinforcement and shear reinforcement. The distances from the shear crack tip to locations D and E are 27 and 42 cm, respectively.

For specimen B₃ right shear span, location F is at the intersection of main shear crack with the longitudinal reinforcement and shear reinforcement. The other loca-

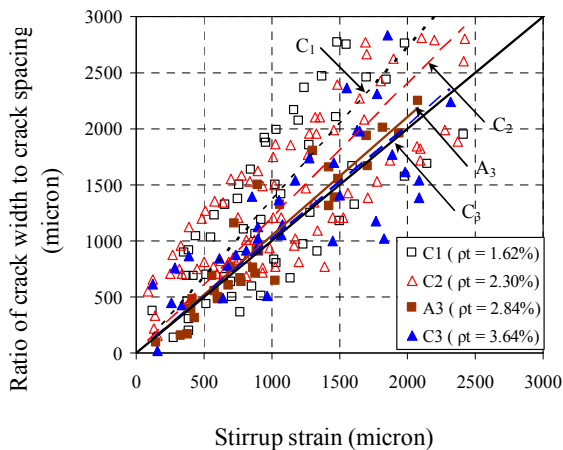
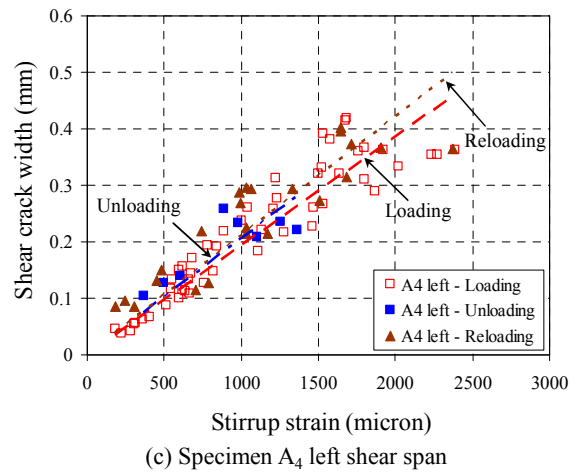
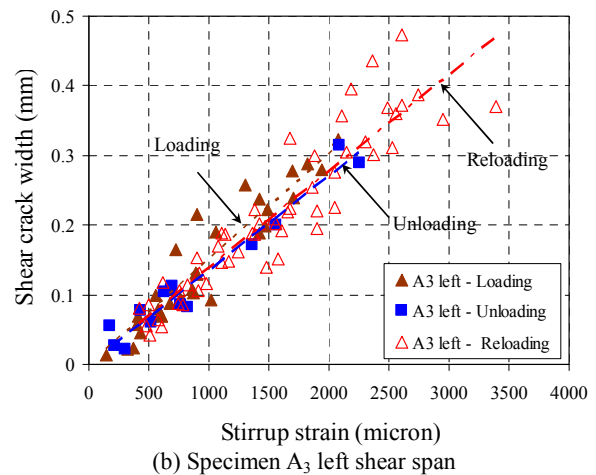
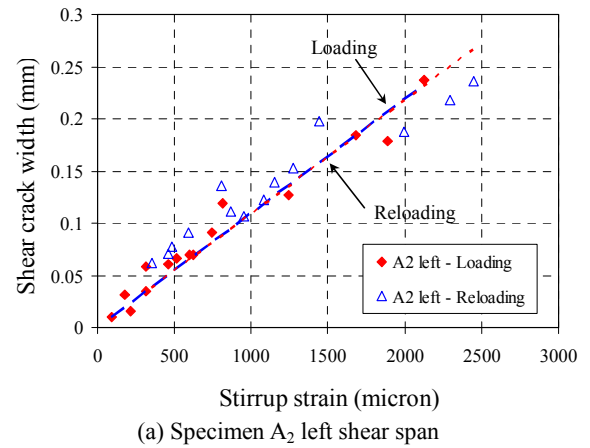


Fig. 20 Effect of longitudinal reinforcement ratio on ratio of crack width to crack spacing-stirrup strain relationship.

Fig. 21 Effect of loading paths on shear crack width-stirrup strain relationship.

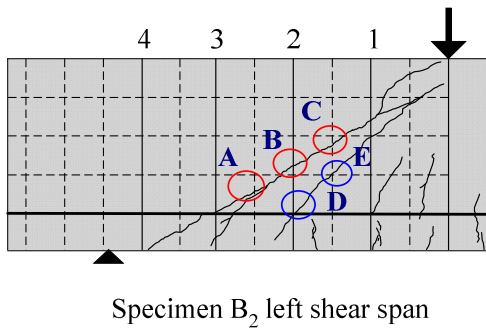


Fig. 22 Measured locations for shear crack width along main shear cracks – Specimen B₂ left shear span.

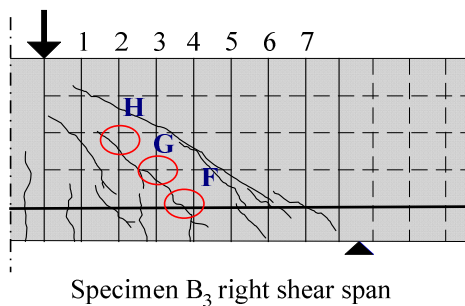


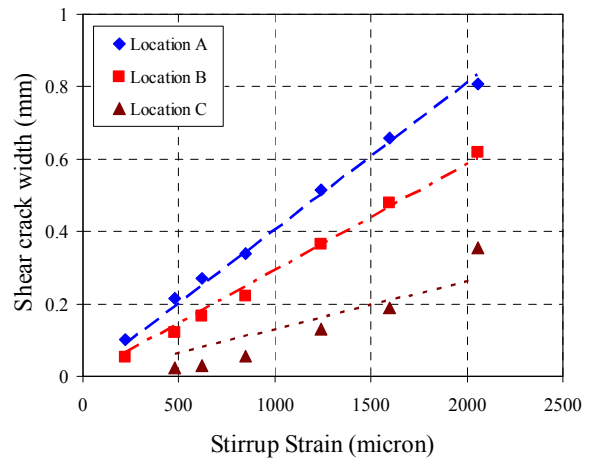
Fig. 23 Measured locations for shear crack width along main shear crack – Specimen B₃ right shear span.

tions (locations G and H) are at the intersection of main shear crack with the shear reinforcement and at a certain distance from intersection of main shear crack with longitudinal reinforcement and shear reinforcement (location F). The distances from the shear crack tip to locations F, G and H are 5, 17 and 30 cm, respectively.

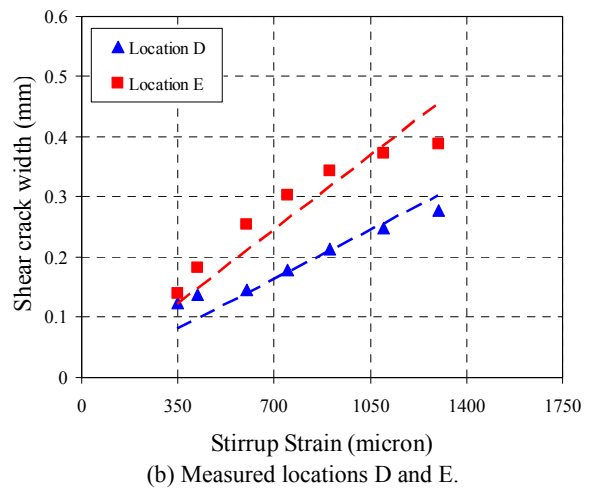
Figures 24 and 25 show the relationship between shear crack width and stirrup strain at different measurement locations for specimens B₂ left shear span and B₃ right shear span, respectively. Figure 24 (a) shows that there is effect of the distance from the shear crack tip. The closer location to the crack tip is the smaller crack width for the same stirrup strain. Also, it can be inferred from Figs. 24 (b) and 25 that the closer location to the intersection of shear crack with longitudinal reinforcement and shear reinforcement is the smaller crack width. Furthermore, it can be observed that the closer to the crack tip is the smaller crack width concerning location H.

It can be concluded that the distance of shear crack from the crack tip and the intersection with the nearest reinforcement (stirrup and longitudinal reinforcement) can significantly affect the variation of shear crack width along the same shear crack.

The variation of shear crack width along shear cracks is considered the reason for the obtained scatter in the shear crack width-stirrup strain relationship.



(a) Measured locations A, B and C.



(b) Measured locations D and E.

Fig. 24 Variation of shear crack width along main shear cracks – Specimen B₂ left shear span.

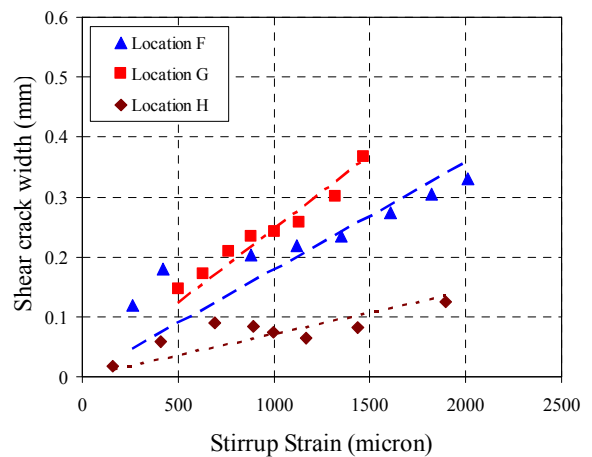


Fig. 25 Variation of shear crack width along main shear crack – Specimen B₃ right shear span.

5. Conclusions

The purpose of the current study is to obtain detailed information on shear cracking behavior in reinforced concrete beams by conducting the experiment of 10 simply supported beam specimens. The experiment investigates the effect of various parameters such as beam size, shear span to depth ratio, side concrete cover to stirrup, stirrup spacing, stirrup configuration, longitudinal reinforcement ratio, and loading paths (loading, unloading and reloading paths) on the spacings between shear cracks and the relationship between shear crack width and stirrup strain at the intersection with shear cracks. Based on the experimental results, the following conclusions can be derived, which are useful information for the development of a rational shear crack displacement prediction method in existing design codes.

- (1) Shear cracks width increases proportionally with both the strain of shear reinforcement and with the spacing between shear cracks, implying that the stirrup strain and diagonal crack spacing are main factors on shear crack displacements.
- (2) It was inferred that the larger beams show greater diagonal crack spacings, and hence result in wider shear crack width in comparison to the smaller beams as the previous studies (Collins *et al.* 2007, Collins and Mitchell 1991 and Shioya *et al.* 1989) show. The reason for such behavior is due to the reduced ability of the longitudinal reinforcement to control the spacings between shear cracks at the mid-height of the larger beams. Near the bottom face of the beam, crack spacing will be controlled by the bond effect of the longitudinal reinforcement; while near to mid-height of the beam, crack spacing will be controlled by the distance from the longitudinal reinforcement. As the distance from mid-height of the beam to the main longitudinal reinforcement increases, the crack spacings become larger.
- (3) The experimental results show that increasing the side concrete cover to stirrup leads to wider diagonal crack spacing and partial absence of shear crack opening control at the surface of the elements. Diagonal crack spacing is considered the main reason causing the difference in crack opening displacements at the same stirrup strain. The increase in diagonal crack spacing can be explained by the increase of concrete area around the stirrup in the case of larger side concrete cover to stirrup. The force which has to be introduced into the surrounding concrete by bond at the end of the transfer length should increase to cause cracking at the surface of RC members. Thus, larger transfer length (or crack spacing) would be needed. The shear crack width at stirrup level is relatively smaller than the shear crack width at the surface of RC member due to crack width control provided by the bond between the stirrup and the surrounding concrete is considered the additional reason.
- (4) It was observed that the larger the stirrup spacing (or smaller stirrup ratio) yields greater shear crack width at the same stirrup strain as the previous studies (Adebar and Leeuwen 1999; Hassan *et al.* 1991; Witchukreangkrai 2006) illustrate. The reason for this behavior is the diagonal crack spacing. The smaller stirrup spacing can significantly reduce the spacings between shear cracks due to decreasing the effective concrete area, in which shear crack width is controlled by the stirrup. Hence, the bond effect between the stirrups and the surrounding concrete would be increased. Increasing the bond effect results in reducing the transfer length (or crack spacing) in which the forces to cause a crack are transferred into the concrete between cracks by the bond stresses. Another possible reason is related to the control of shear crack opening, provided by the bond between the stirrup and the surrounding concrete, which is more effective near the location of stirrup.
- (5) It was found that the closed stirrup configuration shows smaller shear crack width in comparison to the open stirrup configuration at the same stirrup strain as result of the slip of stirrup end which affect the slip at the end of vertical leg of stirrup as proved in the previous study (Hassan and Ueda 1987). The open stirrup configuration is with less efficient hook than that of the closed stirrup configuration. Hence, larger slip between stirrup and concrete occurs, resulting in larger shear crack width at the same stirrup strain.
- (6) Increasing the longitudinal reinforcement amount can better control shear crack opening in the vicinity of longitudinal reinforcement, implying that the longitudinal reinforcement ratio has a significant effect on shear cracking mechanism. The larger amounts of longitudinal reinforcement cause smaller spacings between shear cracks, and thus result in smaller shear crack openings. This can be explained due to the increase in bond effect between the longitudinal reinforcement and the surrounding concrete, thereby enhancing crack control characteristics and reducing the crack spacing. Also, larger amounts of longitudinal reinforcement effectively restrict the widening of flexural cracks and their development into flexure-shear cracks is considered a possible reason.
- (7) The relationship between shear crack width and stirrup strain seems not to be changed among different loading paths (loading, unloading and reloading paths).
- (8) It was observed that the variation of shear crack opening displacements along shear crack is dependent on the distance from the location considered to the crack tip and the intersection with the nearest reinforcement (stirrup and longitudinal reinforcement). In addition, this variation of shear crack width along a shear crack is one of the reasons for the scatter in the observed shear crack width-stirrup

strain ($w-\varepsilon_w$) relationship.

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